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ESTIMATION OF ERROR IN STEADY-STATE TEMPERATURE MEASUREMENT DUE TO CONDUCTION ALONG THE THERMOCOUPLE LEADS

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NOMENCLATURE

A, area of cross-section of the fin [m^2];
A_1–A_6, integration constants;
d, diameter of the fin [m];
$G_A = G_B$, conductance per unit length between body and fin [$W m^{-1} K^{-1}$];
G_C, conductance per unit length between ambient and fin [$W m^{-1} K^{-1}$];
G_j, conductance between junction and fin [$W K^{-1}$];
h, heat transfer coefficient [$W m^{-2} K^{-1}$];
k, thermal conductivity of the fin ($(k_1+k_2)/2$) [$W m^{-1} K^{-1}$];
k_1,k_2, thermal conductivity of the lead wires [$W m^{-1} K^{-1}$];
k_f, thermal conductivity of the filler material [$W m^{-1} K^{-1}$];
k_i, thermal conductivity of the insulation [$W m^{-1} K^{-1}$];
l, length of the sensor along the isothermal path ($L_2 - L_1$);
l_1, major diameter of the insulated sensor leads [m];
l_2, minor diameter of the insulated sensor leads [m];
L_1, length of the sensor in the non-isothermal path in the body [m];
L_2, total length of sensor inside the solid body [m];
m_1, $(G_A/kA)^{1/2}$ [m^{-1}];
m_2, $(G_C/kA)^{1/2}$ [m^{-1}];
m_3, (G_j/kA) [m^{-1}];
n, degree of the polynomial used to approximate the temperature distribution in the body along the embedded sensor length;
P, perimeter of the insulated sensor leads [m];
r, radius of the fin [m];
r_2, equivalent radius of the fin with the insulation [m];
r_h, radius of the hole [m];
r_w, radius of individual thermocouple wires [m];
$S_{A,B,C}$, temperature of the solid, surroundings at various points along thermocouple [K];
S_0, temperature of the solid body at $x = 0$ [K];
S_2, temperature of the solid body between L_1 and L_2 (constant) [K];
S_a, ambient temperature of the fluid [K];
T_0, thermocouple temperature at $x = 0$ [K];
T_1, thermocouple temperature at $x = L_1$ [K];
T_2, thermocouple temperature at $x = L_2$ [K];
X,x, distance along the fin [m].

Greek symbols

α_i, coefficients in the assumed temperature distribution;
β_i, coefficients in the thermocouple temperature distribution;
δ_{eff}, equivalent thickness for the contact resistance [m];
ε, thermocouple error $= T_2 - S_2$;
ε_0, error in the thermocouple when the isothermal path is zero.

1. INTRODUCTION

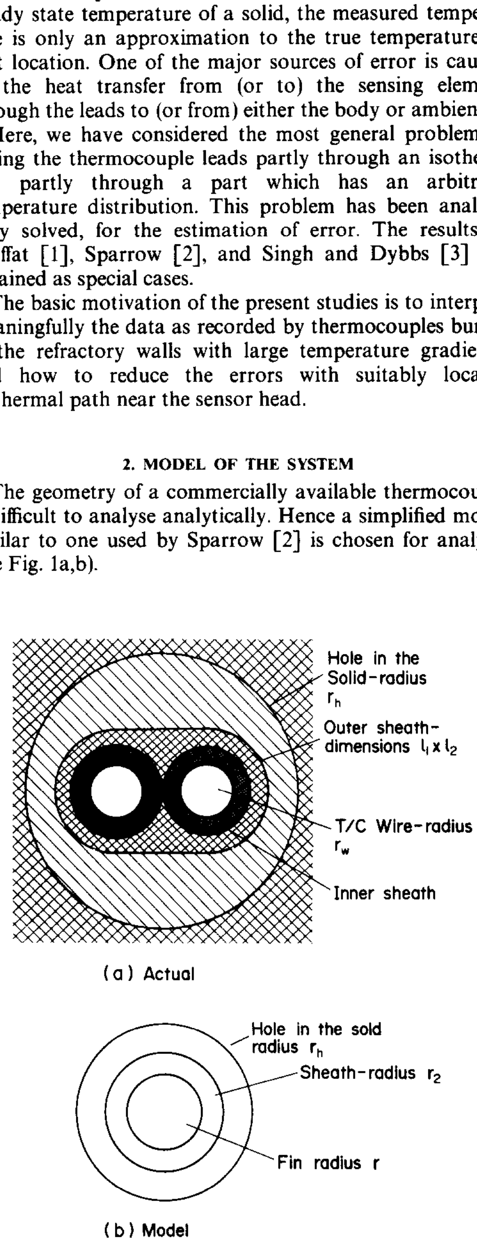
WHEN A temperature sensor is inserted to measure the steady state temperature of a solid, the measured temperature is only an approximation to the true temperature at that location. One of the major sources of error is caused by the heat transfer from (or to) the sensing element through the leads to (or from) either the body or ambient.

Here, we have considered the most general problem of taking the thermocouple leads partly through an isotherm and partly through a part which has an arbitrary temperature distribution. This problem has been analytically solved, for the estimation of error. The results of Moffat [1], Sparrow [2], and Singh and Dybbs [3] are obtained as special cases.

The basic motivation of the present studies is to interpret meaningfully the data as recorded by thermocouples buried in the refractory walls with large temperature gradients, and how to reduce the errors with suitably located isothermal path near the sensor head.

2. MODEL OF THE SYSTEM

The geometry of a commercially available thermocouple is difficult to analyse analytically. Hence a simplified model similar to one used by Sparrow [2] is chosen for analysis (see Fig. 1a,b).



Relations: $r = \sqrt{2}\, r_w$, $r_2 = \frac{l_1 + l_2}{4}$, $r_h = r_h$

FIG. 1. Section across the thermocouple located in the well: (a) actual; (b) model.

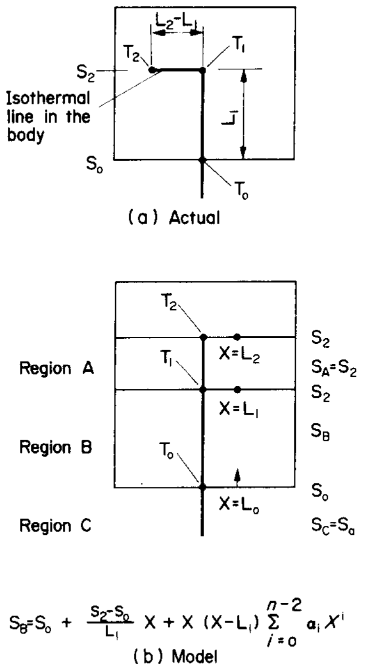


FIG. 2. Configuration of the thermocouple in the (a) actual, (b) model.

Figure 2a represents the actual insertion of the thermocouple in the body and Fig. 2b simulates the model. It is further stipulated that the temperature of the body surrounding the thermocouple can be prescribed.

For the purpose of this analysis, the temperature of the body surrounding the fin is given by:

$$S_A = S_2, \quad \text{for } L_1 \leqslant x \leqslant L_2 \quad \text{(Region A)}$$

$$S_B = S_0 + \frac{S_2 - S_0}{L_1} x + x(x - L_1) \sum_{i=0}^{n-2} \alpha_i x^i, \quad (2.1)$$

$$\text{for } 0 \leqslant x \leqslant L_1 \quad \text{(Region B)}$$

$$S_C = S_a, \quad \text{for } x < 0. \quad \text{(Region C)}$$

The part of the sensor that is inside the solid is surrounded by insulation and filler material. The overall conductance/unit length between the solid and the lateral surface of the sensor can be approximated to be

$$G_A = G_B = \frac{1}{\dfrac{1}{2\pi k_f} \ln \dfrac{r_h}{r_2} + \dfrac{1}{2\pi k_i} \ln \dfrac{r_2}{r}}. \quad (2.2)$$

The contact conductance between the thermocouple junction and the body is difficult to estimate. We can approximate the conductance by

$$G_j = \frac{k_f}{\delta_{\text{eff}}} \pi r^2. \quad (2.3)$$

It has been shown that $\delta_{\text{eff}} = 100$ microns gives satisfactory results [2].

The part of the fin that is exposed to the ambient will exchange heat through convection and radiation. Thus, the overall conductance between sensor's lateral surface and the surroundings may be approximated by

$$G_C = \frac{1}{\dfrac{1}{hP} + \dfrac{1}{2\pi k_i} \ln \dfrac{r_2}{r}}. \quad (2.4)$$

In practical cases h is the function of thermocouple temperature and the thermocouple temperature in turn is decided by h. Hence an iterative procedure is necessary, but an average h may be chosen for analysis.

3. MATHEMATICAL ANALYSIS OF THE PROBLEM

The sensor is divided into three segments, for the purpose of analysis, as shown in Fig. 2b. Region A corresponds to the thermocouple being in an isothermal region, region B where the temperature of the body varies and region C where the thermocouple is in the ambient fluid.

The general heat balance equation for the fin is given by

$$\frac{d^2 T}{dx^2} - m_i^2 (T - S_i) = 0, \quad (3.1)$$

where i takes the values A, B and C and

$$m_A^2 = m_B^2 = G_A/kA = m_1^2$$
$$m_C^2 = G_C/kA = m_2^2. \quad (3.2)$$

The general solutions are:

Region A:

$$T - S_2 = A_1 e^{m_1 x} + A_2 e^{-m_1 x}; \quad (3.3)$$

Region B:

$$T - S_2 = A_3 e^{m_1 x} + A_4 e^{-m_1 x} + \sum_{i=0}^{n} \beta_i x^i, \quad (3.4)$$

where

$$(r+2)(r+1)\beta_{r+2} - m_1^2 \beta_r = -m_1^2 (\alpha_{r-2} - \alpha_{r-1} L_1) \quad \text{for} \quad n \geqslant r \geqslant 2$$

$$2\beta_2 - m_1^2 \beta_0 = -m_1^2 (S_0 - S_2)$$

$$6\beta_3 - m_1^2 \beta_1 = -m_1^2 \left(\frac{S_2 - S_0}{L_1} - \alpha_0 L_1 \right)$$

$$\beta_r = \alpha_{r-2} = 0, \quad \text{for} \quad r > n;$$

Region C:

$$T - S_a = A_5 e^{m_2 x} + A_6 e^{-m_2 x}. \quad (3.5)$$

The constants of integration are determined by the following boundary conditions.

At

$$\begin{array}{ll} x = -\infty, & T = S_a \\ x = 0, & T \text{ and } dT/dx \text{ continuous} \\ x = L_1, & T \text{ and } dT/dx \text{ continuous} \\ x = L_2, & dT/dx = -m_3 (T - S_2), \end{array} \quad (3.6)$$

where $m_3 = G_j/kA$.

The error in temperature measurement after applying the boundary conditions and simplification, may be written as:

$$\varepsilon = -\frac{(S_2 - S_a) + \sum_{i=0}^{n} (m_1 \beta_i L_1^i + i\beta_i L_1^{i-1}) \left(\dfrac{\sinh m_1 L_1}{m_1} + \dfrac{\cosh m_1 L_1}{m_2} \right) - \left(1 + \dfrac{m_1}{m_2}\right) e^{m_1 L_1} \sum_{i=0}^{n} \beta_i L_1^i + \beta_0 + \dfrac{\beta_1}{m_2}}{\left(1 + \dfrac{m_3}{m_2}\right) \cosh m_1 L_2 + \left(\dfrac{m_1}{m_2} + \dfrac{m_3}{m_1} \right) \sinh m_1 L_2}, \quad (3.7)$$

where $\varepsilon = T_2 - S_2$.

Special cases

When the solid body is at a uniform temperature S_2 which is different from the temperature of ambient fluid, then equation (3.7) yields.

$$\frac{T_2-S_2}{S_2-S_a}=\frac{-1}{\cosh m_1 L\left\{\left(1+\frac{m_3}{m_2}\right)+\left(\frac{m_1}{m_2}+\frac{m_3}{m_1}\right)\tanh m_1 L\right\}}, \tag{3.8}$$

which was the result derived by Moffat [1].

In case there is no isothermal section, the error is given by substituting $L_2 = L_1 = L$ in equation (3.7). This result will be the same as derived by Singh *et al.* [3] after algebraic manipulation.

4. DISCUSSION AND RESULTS

The expression for error includes the quantities m_1, m_2, m_3 lengths L_1 and L_2, temperature S_2 and S_a and the temperature distribution in the body. What is striking in the expression is that the numerator does not involve L_2. Consider the quantity $m_1 L_1$ which can be rewritten as:

$$m_1 L_1=\left\{\frac{4}{\left(\frac{1}{2\pi k_f}\ln\frac{r_h}{r_2}+\frac{1}{2\pi k_i}\ln\frac{r_2}{r}\right)k\pi}\right\}^{1/2}\frac{L_1}{d}. \tag{4.1}$$

The term inside the bracket generally is of the order of 1. Thus,

$$m_1 L_1 \approx \frac{L_1}{d}. \tag{4.2}$$

Hence, as it often happens in practice, $m_1 L_1 \gg 1$. Thus,

$$\varepsilon=-\frac{(S_2-S_a)+\sum_{i=0}^{n}(m_1\beta_i L_1^i+i\beta_i L_1^{i-1})\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\frac{e^{m_1L_1}}{2}-\left(1+\frac{m_1}{m_2}\right)e^{m_1L_1}\sum_{i=0}^{n}\beta_i L_1^i+\left(\beta_0-\frac{\beta_1}{m_2}\right)}{\left(1+\frac{m_3}{m_2}+\frac{m_3}{m_1}+\frac{m_1}{m_2}\right)\frac{e^{m_1L_1}}{2}e^{m_1 l}}, \tag{4.3}$$

where $L_2 = L_1 + l$, where l represents the length along the isothermal path. Then the error can now be rewritten as:

$$\varepsilon=\varepsilon_0 e^{-m_1 l}, \tag{4.4}$$

where ε_0 denotes the error when $l = 0$ corresponding to the situation when there is no isothermal path. Thus with the introduction of the isothermal path the error falls exponentially with l. Since $m_1 l \approx l/d$, the ratio l/d has to be chosen large. Normally as a thumb rule one selects l/d approximately of the order of 10, which ensures that error is reduced by a factor of 10^4.

In case it is not possible to provide a long isothermal path, the next problem is to estimate ε_0. For this, the temperature distribution along the sensor should be known. It is possible to determine this by using several thermocouples at different depths of immersion and estimate the temperature field using an iterative procedure by employing the corrected values of the thermocouple for predicting the temperature distribution and vice-versa, until convergence is obtained. In order to illustrate the effect of the temperature distribution of the body on ε_0, the following special cases may be considered:

(i) Constant temperature S_2:

$$\varepsilon_0=-\frac{(S_2-S_a)e^{-m_1L_1}}{2\left(1+\frac{m_1}{m_2}+\frac{m_3}{m_2}+\frac{m_3}{m_1}\right)}; \tag{4.5}$$

(ii) Linear temperature distribution with the outside surface temperature equal to ambient temperature S_a:

$$\varepsilon_0=-\left\{\frac{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}{2\left(1+\frac{m_1}{m_2}+\frac{m_3}{m_1}+\frac{m_3}{m_2}\right)}\right\}\frac{S_2-S_a}{L_1}; \tag{4.6}$$

(iii) Quadratic temperature distribution with the outside surface temperature equal to ambient temperature S_a:

$$\varepsilon_0=-\left\{\frac{\frac{1}{m_1}+\frac{1}{m_2}}{2\left(1+\frac{m_1}{m_2}+\frac{m_3}{m_2}+\frac{m_3}{m_1}\right)}\right\}\left(\frac{S_2-S_a}{L_1}+\alpha_0 L_1\right). \tag{4.7}$$

Thus, even though the error ε_0 falls exponentially with L_1 for case (i), (this also corresponds to the case of an isothermal path) in case (ii) and (iii) the error decreases only as the average temperature gradient. If the temperature gradient near the junction is small (corresponding to α_0 being negative), the error is also small.

Summarising, the advantage of isothermal portion can be clearly seen in decreasing the error in thermocouple measurement and should be incorporated since this eliminates the problem of estimating.

5. COMPARISON WITH EXPERIMENTS

To check the validity of the model some experiments were performed using Sillimanite as the solid refractory material and Chromel–Alumel as the thermocouple sensors. The experimental set up is shown in Fig. 3. The temperature profile in the test piece was linear as verified by inserting auxilliary thermocouples at various points. The surface temperature S_0 is determined by extrapolating this data. The theoretical estimates are made on the basis of the following data:

$k_1 = 19.25\,\mathrm{W\,m^{-1}\,K^{-1}}$ (Chromel) $l_1 = 1.6 \times 10^{-3}\,\mathrm{m} \pm 0.02 \times 10^{-3}\,\mathrm{m}$
$k_2 = 29.72\,\mathrm{W\,m^{-1}\,K^{-1}}$ (Alumel) $l_2 = 2.5 \times 10^{-3}\,\mathrm{m} \pm 0.02 \times 10^{-3}\,\mathrm{m}$
$k_i = 0.26\,\mathrm{W\,m^{-1}\,K^{-1}}$ $r_h = 1.25 \times 10^{-3}\,\mathrm{m} \pm 0.02 \times 10^{-3}\,\mathrm{m}$
$k_f = 0.045\,\mathrm{W\,m^{-1}\,K^{-1}}$ $L_1 = 21 \times 10^{-3}\,\mathrm{m} \pm 0.5 \times 10^{-3}\,\mathrm{m}$
$h = 100\,\mathrm{W\,m^{-2}\,K^{-1}}$ $L_2 = 41 \times 10^{-3}\,\mathrm{m} \pm 0.5 \times 10^{-3}\,\mathrm{m}$
$r_w = 0.37 \times 10^{-3}\,\mathrm{m}$ $\delta_{\mathrm{eff}} = 0.1 \times 10^{-3}\,\mathrm{m}$.

Since there was no filler material k_f is taken as that of air. An average value of h was chosen. However, it was found that the estimation of error is insensitive to h. Some typical results are presented in Table 1. It gives the measured temperature recorded by thermocouples A and B and the true temperature after applying the corrections. It can be seen that the estimated temperature of both the thermocouples are quite close thus justifying the validity of the proposed model.

Table 1

No.	Surface temperature S_0 K	Thermocouple A: Measured temperature K	Thermocouple A: Correction K	Thermocouple A: True temperature K	Thermocouple B: Measured temperature K	Thermocouple B: Correction K	Thermocouple B: True temperature K
1	530	976	119	1095	1088	3	1091
2	550	894	101	995	1000	2	1002
3	470	703	70	770	768	1	769
4	495	753	62	815	796	2	798

Experimental error in temperature measurement was within $\pm 0.5\%$ including thermocouple and recorder errors.
In the case of thermocouple B, as can be seen, the correction is less than the measurement error.

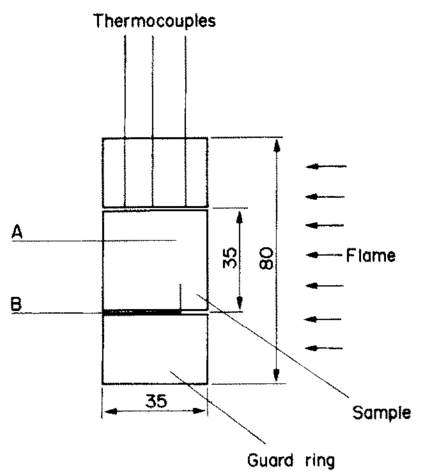


FIG. 3. Schematic diagram of the experimental arrangement: A. Thermocouple perpendicular to isothermal path; B. Thermocouple along (part) isothermal path.

6. CONCLUSION

This paper presents the analysis for estimating the error in the steady state temperature measurement due to conduction along the thermocouple leads when they are placed partly along an isothermal path and partly along a direction with an arbitrary temperature distribution. The validity of the model has been checked with some experimental data.

Using the expression for the error developed in the paper, it is possible to estimate the error for a given experimental condition, or alternatively the experiment can be designed in such a way that the error in measurement is within reasonable limits.

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